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FOLIATIONS WITHOUT COMPACT LEAVES ON SOME PSEUDO-ANOSOV BUNDLE

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Abstract. In this paper we continue, as in [4], the investigation on pseudo-Anosov bundles of the classification of foliations without compact leaves.

1. Introduction

Let ψ be an Anosov diffeomorphism of the torus T^2 induced by a matrix $A \in SL(2,\mathbb{Z})$ with $|trA| > 2$. E. Ghys and V. Sergiescu (see[1]) built the 3-manifold $\mathbb{T}^3_A$ by suspension of ψ and classified all $C^r(r \geq 2)$ codimension one foliations without compact leaves.

Let Σ be the genus 2 closed surface. Using the method of branched covering kindly chosen, we obtained from ψ a pseudo-Anosov diffeomorphism ϕ of Σ. The $3-$manifold V^3_ϕ is built by suspension of ϕ. Unlike $\mathbb{T}^3_A$, there is no completed theorem of classification of codimension one foliations without compact leaves on V^3_ϕ. If $\mathcal{F}$ is a foliation without compact leaves in a pseudo-Anosov bundle V, the main difficulty to get a completed classification theorem is because it is difficult as in an Anosov bundle to get a fiber S so that the singular foliation $\mathcal{F}|(S \times \{0\})$ coincides with $\mathcal{F}|(S \times \{1\})$ and to classify the foliation of $\Sigma \times [0,1]$.

There are partial results in this direction. For example H. Nakayama (see[2]) classified up to covering, all transversely affine foliations without compact leaves and in the Euler class of the fibration.
H. Dathe (see[3]) proved that all transversely projective foliations close enough to a model foliation are conjugated to this model.
In a recent work (see [4]) we give a partial classification of transversely projective foliations on V^3_ϕ. In the present paper we analyse the classification of foliations without compact leaves on V^3_ϕ in the Euler class of the fibration.

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Our paper is organised as follow. In the first two paragraphs we give the construction of model foliation on Anosov bundle and model foliations on pseudo-Anosov bundle to understand the topics.

In the last paragraph we analyse the problem of classification of foliations without compact leaves on V^3_ϕ.

2. Foliations without compact leaves on $\mathbb{T}^3_A$

2.1. **Construction of $\mathbb{T}^3_A$.** Let $\psi : T^2 \to T^2$ be an Anosov diffeomorphism of the torus T^2 induced by a hyperbolic matrix A (i.e $A \in SL(2, \mathbb{Z})$ and $|trA| > 2$). We built by suspension of ψ the 3-manifold:

$$\mathbb{T}^3_A = T^2 \times I/(x, 0) \sim (\psi(x), 1)$$

with $I = [0, 1]$. This manifold is a torus bundle over the circle called Anosov bundle.

2.2. **Model foliations on $\mathbb{T}^3_A$.** The matrix A has two eigenvalues λ and $1/\lambda$. The foliations on $\mathbb{R}^2$ by lines of slope λ and $1/\lambda$ are invariant by the action of $\mathbb{Z}^2$ and induce on the torus T^2 two foliations invariant by ψ.

By suspension of these foliations of T^2 one obtains on $\mathbb{T}^3_A$ two foliations which we call model foliations.

2.3. **Classification.** E. Ghys and V. Sergiescu ([1]) classified all $C^r (r \geq 2)$ codimension one foliations without compact leaves. Their result is as follow: ..

Theorem 2.1. [1] *Every codimension one C^r $r \geq 2$ transversely affine orientable foliation without compact leaves on the genus one orientable hyperbolic bundle is C^{r-2}-conjugate to a model foliation.*

3. Foliations without compact leaves on V^3_ϕ

3.1. **Construction of V^3_ϕ.** Let ψ be a diffeomorphism of the torus T^2 induced by the matrix:

$$\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$$

The set of fixed points of ψ is:

$$Fix(\psi) = \{(0, 0), (1/5, 2/5), (2/5, 4/5), (3/5, 1/5), (4/5, 3/5)\}$$

Set

$$T = T^2 - \{(1/5, 2/5), (4/5, 3/5)\}.$$

The open manifold T retracts on a bunch of three circles $\{\alpha, \beta, \varepsilon\}$, $\pi_1(T)$ is a free group generated by α, β and ε, where α and β are the generators of $\pi_1(T^2)$ and ε surrounds the hole $(1/5, 2/5)$. Let $q : S \to T$ be the regular covering of T corresponding to $Ker\mu$ where:

$$\mu : \pi_1(T) \to \mathbb{Z}/2\mathbb{Z}$$

is the representation sending α, β and ε on 1. The manifold S is open and diffeomorphic to a closed genus 2 surface Σ perforated in two points. The covering q extends to a branched covering $p : \Sigma \to T^2$ with branched set of points $\{(1/5, 2/5), (4/5, 3/5)\}$. The diffeomorphism ψ lifts by p to a pseudo-Anonsov diffeomorphism ϕ of Σ. One defines the suspension 3-manifold of ϕ by:

$$V_\phi^3 = \Sigma \times I/(x, 0) \sim (\phi(x), 1).$$

This fiber bundle is analogous in higher genus to $\mathbb{T}_A^3$. It is a surface bundle over the circle called pseudo-Anosov bundle.

3.2. **Model foliations on V_ϕ^3.** V_ϕ^3 is an example of pseudo-Anososv bundle obtained by suspension of a pseudo-Anososv diffeomorphism. We recall in this section the general construction of model foliations on any pseudo-Anosov bundle. The model foliations on V_ϕ^3 are obtained in the same way.

Let Σ be a genus g closed orientable surface with $g > 1$ and $\phi : \Sigma \longrightarrow \Sigma$ a diffeomorphism.

Definition 3.1. *A diffeomorphism $\psi : \Sigma \to \Sigma$ is called pseudo-Anosov with 4 branches if there are two foliations $\mathcal{F}^s$ and $\mathcal{F}^u$ on Σ with the following properties:*
i) $\mathcal{F}^s$ and $\mathcal{F}^u$ are transversely oriented and measured. $\mathcal{F}^s$ and $\mathcal{F}^u$ are singular with saddles singularities with four separators, and have the same set of singularities K and are transverse on $\Sigma \backslash K$.
ii) There is some constant $\lambda > 1$ such that:

$$\phi^*(\mathcal{F}^s, \mu^s) = (\mathcal{F}^s, \lambda^{-1}\mu^s)$$
$$\phi^*(\mathcal{F}^u, \mu^u) = (\mathcal{F}^u, \lambda\mu^u)$$

The manifold

$$M = \Sigma \times I/(x, 0) \sim (\psi(x), 1)$$

is called pseudo-Anosov bundle.

The foliations $\mathcal{F}^s$ and $\mathcal{F}^u$ are called respectively the stable foliation and the unstable foliation of ψ and λ is the dilation number.

Example 3.1. *Each genus $g > 1$ closed orientable surface Σ is the total space of a two sheeted branched covering over the torus T^2 with branched set of points K such that card$K = 2g - 2$. Indeed, piercing the torus T^2 throughout $n = $ card$K$ points; this open surface retracts on a bunch of $n+1$ circles ξ_1,...,ξ_{n+1}, where each ξ_1,...,ξ_{n-1} surrounds a hole while ξ_n and ξ_{n+1} generate $\pi_1(T^2)$. The last hole noted ∞ is homologically surrounded by $[\xi_1] + ... + [\xi_{n-1}]$. We build a covering associated to the homomorphism $\pi_1(T^2 \backslash K) \to \mathbb{Z}/2\mathbb{Z}$ which sends $\xi_1, ..., \xi_{n-1}$ on 1 and take any value on ξ_n and ξ_{n+1}.Thus the compactification gives a covering $p : \Sigma \to T^2$ which is branched at any point of K.*

Let ϕ be a linear Anosov diffeomorphism of T^2 which is identity on K and lifts up to ψ on Σ. The stable foliation of ϕ is transversely oriented and its singularities have 4 branches. So ψ is pseudo-Anosov and its stable foliation is defined as a measured foliation by a 1$-$form ω^s such that $\psi^*\omega^s = \lambda\omega^s$ ($\lambda \in \mathbb{R}, \lambda > 1$). Also we have $\psi^*\omega^u = \lambda^{-1}\omega^u$ where ω^u defines the unstable foliation of ψ.

Remark 3.1. *As $\psi : \Sigma \to \Sigma$ is the lift by p of $\phi : T^2 \to T^2$ so the stable foliation of ψ is the lift of the stable foliation of ϕ. Also the unstable foliation of ψ is the lift by p of the unstable foliation of ϕ.*

We'll build on M model foliations by proceeding as on V.

On M note by ω^s and ω^u the closed $1-$forms defining respectively the stable foliation and the unstable foliation of ψ and λ the dilation number of ψ. We can define the suspension foliations of the stable foliation and the unstable foliation of ψ. On $\Sigma \times I$, let the $1-$form $\Omega_\sigma = \lambda^{\epsilon(\sigma)t}\omega^\sigma + dt$ where $\sigma = s$ or $\sigma = u$ and $\epsilon(\sigma) = 1$ if $\sigma = s$ and -1 if $\sigma = u$. The $1-$form Ω_σ defines a foliation H^σ on M with a finite number of contact circles $\gamma_1, \gamma_2, ..., \gamma_n$ with the fibration of M over S^1. Each circle γ_i has a tubular neighborhood V_i foliated¨ as in figure 1.

Cutting the V_i we obtain a compact $3-$manifold W and H^σ induces on the boundary of W four plane Reeb components R_i^j, $j = 1, ..., 4$ which are pairwise parallels. Replace V_i by a neighborhood U_i with a foliation obtained by opening along γ_i and gluing each leaf of R_i^j with a leaf of the opposite side. This process is the desingularization. Precisely let S be the common singular finite set to ω^s and ω^u, on $(\Sigma \backslash S) \times \mathbb{R}$ the $1-$forms $\lambda^t\omega^s + dt$ and $\lambda^{-t}\omega^u + dt$ define two nonsingular foliations H^s and H^u on $M \backslash S'$ where $S' = \cup\gamma_i$ which have the same Euler class as the fibration. The neighborhood V_i of each γ_i of S' is replaced by a neighborhood W_i with a foliation ν^σ $(\sigma = s, u)$ as figure 2.

The foliation H^σ extends to a transversely affine foliation σ on M with the same Euler class as the fibration. We call these foliations model(positive) foliations of M. It is also possible to build model(negative) with opposite Euler class as the fibration.

Indeed consider a fiber of the fibration π and the restriction $\mathcal{F}^\sigma$. The orientation of the foliation at the singularities is the same or opposite to the orientation of the tangent space of the fiber. We suppose that the orientation is the same outside singularities. We say that a singularity is positive if the two orientations are the same and negative otherwise. If all the singularities are positive then the Euler classes are the same (see[3])and opposite otherwise.

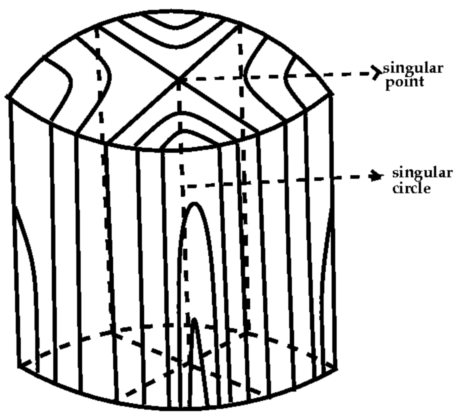

FIGURE 1. tubular neighborhood V^i

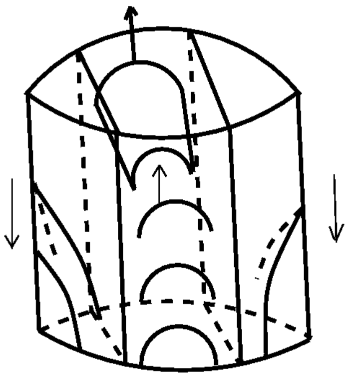

FIGURE 2. Tubular neighborhood (W_i, ν^σ)

We obtained in particular on V_ϕ^3 two foliations without compact leaves in the Euler class of the fibration. These foliations are the model foliations on V_ϕ^3.

3.3. **The problem of classification of foliations in pseudo-Anosov bundle.** The manifold V_ϕ^3 satisfied the hypothesis of the following Nakayama theorem ([2]):

Theorem 3.1. [2] *Let Σ be a closed orientable surface with genus greater than 1 and let $\pi : M \to S^1$ be an oriented $\Sigma-$bundle over S^1 of pseudo-Anosov type such that the real eigenvalues of its monodromy matrix are λ and $\frac{1}{\lambda}$, and the eigenspace with respect to λ (resp. $\frac{1}{\lambda}$) is one dimensional, where $\lambda(> 1)$ is the dilation number of M. Suppose that $\mathcal{F}$ is transversely oriented and transversely affine codimension one foliation of M without compact leaves satisfying the Euler class equality $\chi(T\mathcal{F}) = \pm\chi(T\pi)$ $(\in H^2(M, \mathbb{Z}))$,. Then there is a finite covering of $\mathcal{F}$ which is C^0 isotopic to a suspension foliation of a pseudo-Anosov diffeomorphism.*

H. Dathe proved the following result on V_ϕ^3:

Theorem 3.2. [3] *The model foliations on V_ϕ^3 do not have a transversely projective perturbation which is not affine.*

Then he deduced the following corrolary:

Corollary 3.1. [3] *Any transversely projective foliation on V_ϕ^3 enough closed to a model foliation is conjugate to this model.*

At last he asked this question:
Question:[3] *Is any tranversely projective foliation without compact leaves in the Euler class of a model on V_ϕ^3, conjugate to this model foliation?*
In [4] we proved the following result:

Theorem 3.3. [4] *Let $\mathcal{F}$ be a tranversely projective foliation without compact leaves on V_ϕ^3 in the Euler class of the fibration. Then it is up to a finite covering C^0-isotopic to a model foliation.*

We ask the following questions on V_ϕ^3:
1. Is there a metric g in V_ϕ^3 such that all leaves are minimal surfaces?
2. Is there a harmonic $1-$form for g cohomologous to $\pi^* d\theta$ such that the fibers are minimal surfaces.

Before examining these questions we need to recall the following results in general theory of three dimensional manifolds.
Thurston proved the following result:

Theorem 3.4. *Let $\mathcal{F}$ be a foliation without compact leaves on a $3-$dimensional manifold M. If Σ is a closed surface in M such that there is an injection of $\pi_1(\Sigma)$ in $\pi_1(M)$, then there exists Σ' isotopic to Σ such that the contact of Σ' with $\mathcal{F}$ are saddle singularities.*

J. Hass proved the following result in [5]:

Theorem 3.5. [5] *Let M be an oriented closed $3-$manifold and let $\mathcal{F}$ be a codimension one foliation of M. M admits a metric in which each leaf of $\mathcal{F}$ is a minimal surface if and only if every compact leaf of $\mathcal{F}$ intersects some closed curve which is transverse to $\mathcal{F}$. In such a metric, each leaf is homology area minimizing.*

R. Schoen and S.T. Yau proved in [8] the following theorem:

Theorem 3.6. [8] *Suppose N is a compact Riemannian manifold, M_0 a compact surface of genus $g > 1$, and $\phi : M_0 \to N$ is a map which is injective on fundamental groups. There exists a conformal structure on M_0 and a branched minimal immersion $f : M_0 \to N$ whose action on π_1 is conjugate to that of ϕ, and which minimizes area over all such maps.*

3.4. **Foliations without compact leaves in V_ϕ^3.** According to J. Hass result [5] the first question is trivial. Indeed, the pseudo-Anosov bundle V_ϕ^3 is a compact manifold and if the foliation is without compact leaves, it is a taut foliation and there is a closed transversal which intersects all the leaves of the foliation. Appliying J. Hass theorem, V_ϕ^3 admits a metric such that all leaves are minimal surfaces.
In the perspective of the second question, we prove the following result.

Theorem 3.7. *For any fiber Σ of V_ϕ^3 there is an immersion of fundamental group $\pi_1(\Sigma)$ in $\pi_1(V_\phi^3)$.*

Proof
Recall that if Σ is a fiber of V_ϕ^3 then it's fundamental group is given by

$$\pi_1(\Sigma) = \langle a, b, c, d \quad | \quad [a,b][c,d] = Id\rangle.$$

Also it is known from [3] that the fundamental group of $\pi_1(V_\phi^3)$ is generated by $\pi_1(\Sigma)$ and an element t with the following relations:

$$
\begin{aligned}
\phi_* a &= tat^{-1} \\
\phi_* b &= tbt^{-1} \\
\phi_* c &= tct^{-1} \\
\phi_* d &= tdt^{-1} \\
[a,b][c,d] &= Id
\end{aligned}
\tag{1}
$$

If Σ is the fiber then the fundamental groups fit to a short exact sequence:

$$1 \to \pi_1(\Sigma) \to \pi_1(V_\phi^3) \to \mathbb{Z} \to 1$$

This implies a surjective homomorphism from $\pi_1(V_\phi^3)$ to $\mathbb{Z}$, with the kernel being isomorphic to $\pi_1(\Sigma)$. This injects the fundamental group of the fiber Σ into the fundamental group of the total space V_ϕ^3. We can construct the immersion as follows.

As Σ is the fiber of V_ϕ^3, we have the injection $i : \Sigma \to V_\phi^3$ and the map $i_\sharp : \pi_1(\Sigma) \to \pi_1(V_\phi^3)$, such that $i_\sharp(a) = a$, $i_\sharp(b) = b$, $i_\sharp(c) = c$ and $i_\sharp(d) = d$, is injective. This prove that there is an immersion of $\pi_1(\Sigma)$ in $\pi_1(V_\phi^3)$. □

Using this Theorem and the result of R. Schoen and S.T. Yau [8], we obtained the following result:

Theorem 3.8. *For any fiber Σ of the fibration $p : V_\phi^3 \to S^1$, there exists an immersion $h : \Sigma \to V_\phi^3$ so that $h_\sharp = i_\sharp$ on $\pi_1(\Sigma)$ and the induced area of h is least among all maps with the same action on $\pi_1(\Sigma)$.*

Proof

Let Σ be a fiber of the fibration. According to the previous theorem, there exists an injection of Σ on V_ϕ^3 such that the map $i_\sharp : \pi_1(\Sigma) \to \pi_1(V_\phi^3)$ defined as above is injective. According to [8], there exists a minimal immersion $h : \Sigma \to V_\phi^3$ so that $h_\sharp = i_\sharp$ on $\pi_1(\Sigma)$ and the induced area of h is least among all maps with the same action on $\pi_1(\Sigma)$. □

Remark 3.2. *Using Thurston result we can say that if $\mathcal{F}$ is a foliation without compact leaf on V_ϕ^3 and Σ is a fiber, then there exists Σ' isotopic to Σ such that the contact of Σ' with $\mathcal{F}$ are saddle singularities.*

Now, we can ask if we can construct a harmonic form on V_ϕ^3 cohomologous to $p^*d\theta$ such that the fibration is mnimal? We first analyse harmonic closed one form in V_ϕ^3 and the relation between the harmonic closed one form in the fiber. Failing to find an answer to this question, we conjecture that the answer will be negative using a result in [10].

3.5. **Foliation defined by closed harmonic form.** Consider the canonical angular form $d\theta$ on the circle S^1, where $\theta = \frac{1}{2\pi}\phi$ where ϕ is the angle, a multivalued function on the circle, $d\theta$ is a closed form having no zero.
A closed $1-$form on a manifold can be obtained as follows. Let $f : M \to S^1$ be a smooth map. The pullback $\omega = f^*d\theta$ is a closed $1-$form on M. The zero of ω are nondegenerate if and only if f has only Morse critical points.

Varying the circle-valued map $f : M \to S^1$, we obtain a variety of induced closed $1-$form $\omega = f^*d\theta$.

Lemma 3.1. [7] *A closed* $1-$*form* ω *on a smooth manifold* M *can be represented as* $\omega = f^*d\theta$ *where* $f : M \to S^1$, *is a smooth map, if an only if the de Rham cohomology class* $\xi = [\omega] \in H^1(M, \mathbb{R})$ *of* ω *is integral i.e* $\xi \in H^1(M, \mathbb{Z})$.

Definition 3.2. *A closed* $1-$*form* ω *is called generic if each singular leaf of the foliation defined by* ω *contained precisely one zero of* ω.

Lemma 3.2. [7] *Given a closed* $1-$*form* ω *with Morse-type zeros, then there exists a small perturbation* ω', *which is generic and has the same cohomology class* $[\omega'] = [\omega] = \xi$ *and* $\omega|U = \omega|U$, *where* $U \subset M$ *is an open subset of* M *containing all zeros of* ω.

Definition 3.3. 1. *A closed* $1-$*form is called Morse if all its zeros are nondegenerate.*
2. *A closed* $1-$*form* ω *on a smooth manifold* M *is called intrinsically harmonic if it is harmonic with respect to some Riemannian metric on* M.
3. *A closed* $1-$*form* ω *is called transitive if for any point* $p \in M$, *which is not a zero of* ω, *there exists a closed* $\omega-$*positive loop* $\gamma : [0, 1] \to M$, *which starts and ends at* p.

Theorem 3.9. *(E. Calabi)* [7]
Let M *be a closed smooth manifold. A closed* $1-$*form on* M *having Morse type zeros is intrinsically harmonic if and only if it is transitive.*

We obtain the following result:

Theorem 3.10. *The pullback of the nonsingular harmonic closed* $1-form$ $\omega = p^*d\theta$ *is a closed singular harmonic closed* $1-form$ *on the fiber of the fibration* $p : V^3_\phi \to S^1$.

Proof
Let $i : \Sigma \to V^3_\phi$ be the injection of the fiber Σ in V^3_ϕ. Le $p : V^3_\phi \to S^1$ be the fibration on the circle. Consider the canonical angular form $d\theta$ on the circle S^1, where $\theta = \frac{1}{2\pi}\phi$ where ϕ is the angle, a multivalued function on the circle, $d\theta$ is a closed form having no zero. The pullback $\omega = p^*d\theta$ is a closed $1-$form having no zero on V^3_ϕ. $\alpha = i^*\omega = i^*p^*d\theta$ is a closed one form on Σ. The form α is a singular closed one form because a genus g surafce with $g > 1$ can not admit a nonsingular foliation. If α is a closed nonsingular one form on Σ it will define a nonsingular foliation on the surface which is a contradiction of the fact that the Euler class of Σ is $\xi(\Sigma) = 2-2g$ is not zero. □

Remark 3.3. *Now consider the singular foliation on the fiber* Σ *defined by the singular form* $\alpha = i^*\omega$. *As the form* α *is a Morse type one form, the foliation* $\mathcal{F}_\alpha$ *has a finite number of singularities.*

On V^3_ϕ *the nonsigular foliation defined by the harmonic form* $\omega = p^*d\theta$ *is a codimension one foliation on the fiber bundle. Note by* ω^s *and* ω^u *the closed* $1-$*forms defining respectively the stable foliation and the unstable foliation of* ψ *and* λ *the dilation number of* ψ. *We consider the suspension foliations* *of the stable foliation and the unstable foliation of* ψ*. On* $\Sigma \times I$*, let the* $1-$*form* $\Omega_\sigma = \lambda^{\epsilon(\sigma)t}\omega^\sigma + dt$ *where* $\sigma = s$ *or* $\sigma = u$ *and* $\epsilon(\sigma) = 1$ *if* $\sigma = s$ *and* -1 *if* $\sigma = u$*. The* $1-$*form* Ω_σ *defines a foliation* H^σ *on* V^3_ϕ *with a finite number of contact circles* $\gamma_1, \gamma_2, ..., \gamma_n$ *with the fibration of* V^3_ϕ *over* S^1*.*

We can ask, if the foliation $\mathcal{F}_\sigma$ *is conjugate to one of the stable foliation or the unstable one. Another question which can arise is to know if the foliation on* V^3_ϕ *defined by the harmonic form* ω *is conjugate to one of the model foliation, or is it the suspension of the foliation* $\mathcal{F}_\alpha$*. We can ask if the foliation* $\mathcal{F}_\alpha$ *can be suspended to a foliation on the fiber bundle.*

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Let $\mathcal{F}$ be a foliation without compact leaves on V^3_ϕ and let g be a metric such that the leaves are minimal. Is there a harmonic one form cohomologous to $p^*d\theta$ such that the fibration is minimal? This means that the fibration is a foliation such that the leaves are compact minimal surfaces of genus $g > 1$. We are not able to construct this form but we wish that this will have a negative unswer for the following reason. In [10] the following result is proven.

Theorem 3.11. [10]
Let M *be a three-dimensional closed hyperbolic manifold. Then there does not exist a geometric foliation of* M *by closed minimal surfaces of genus* $g > 1$.

It is known that the interior of a fiber bundle over the circle has a hyperbolic structure. Combining this and the previous theorem, we wish that the unswer will be negative.

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