



Wasan geometry with the division by 0

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Abstract. Results in Wasan geometry of tangents circles can still be considered in a singular case by the division by 0.

1. INTRODUCTION

Japanese mathematics in Edo period is called Wasan. Wasan geometry considers some relationships which arise when some elementary figures such as lines and circles get together. The result does not consider the degenerate case explicitly where the circles are lines or points. This is a singular case for the parameters expressing the circles. In this paper we show that we can still consider such a case with the definition of the division by 0. We also show that we can consider by manipulating equations of the circles with no consideration of limit and the result obtained in the ordinary case is still true in the singular case.

2. THE DIVISION BY 0

In this section we give a brief introduction of the division by 0. Let us consider the function $F : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ satisfying

$$(i) F(a, b) = a/b, \text{ if } b \neq 0, \quad (ii) F(a, b)F(c, d) = F(ac, bd).$$

Obviously F is a generalization of the ordinary fraction a/b . While $F(a, 0) = F(a, 0)(2/2) = F(2a, 0) = F(a, 0)(2/1) = 2F(a, 0)$ gives $F(a, 0) = 0$. Hence we define as follows:

$$(d1) a/0 = 0 \text{ for any real number } a \text{ [2].}$$

Notice that $F(a, 0) = 0$ is obtained in any field with characteristic different from 2.

Any circle or any line has an equation $S(x, y) = a(x^2 + y^2) + 2gx + 2fy + c = 0$. If $S(x, y) = 0$ expresses a circle, its radius is given by

$$R = \sqrt{\frac{g^2 + f^2 - ac}{a^2}}.$$

This implies $R = 0$, if $a = 0$ by (d1). Therefore we define as follows:

$$(d2) \text{ If we consider a line as a circle, its radius equals } 0 \text{ [5].}$$

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Remark 1. We get $\tan \pi/2 = 0$ by (d1), i.e., the slope of a perpendicular equals 0 if we assume the definition of the division by 0. Therefore we can consider that the orthogonality and the tangency are the same [3].

3. THE PROPOSITION

Generalizing a problem in Wasan geometry in [1], we get the following proposition (see Figure 1).

Proposition 1 ([4]). *Let α , β , γ be circles of radii a , b , c , respectively. If s and t are tangents of β parallel to each other, α touches s from the same side as β and β externally, and γ touches t from the same side as β and α and β externally, then the following relation holds:*

$$(1) \quad c = \frac{b^2}{4a}.$$

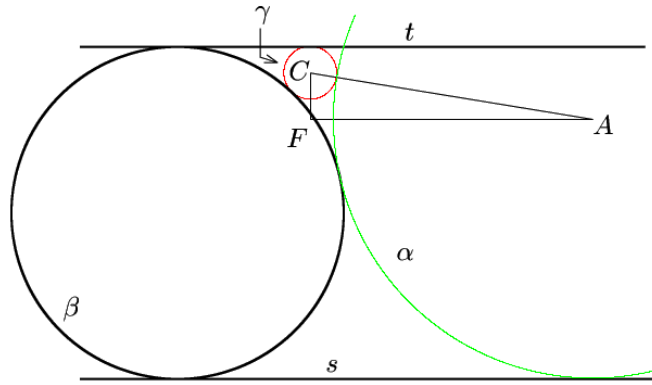


Figure 1.

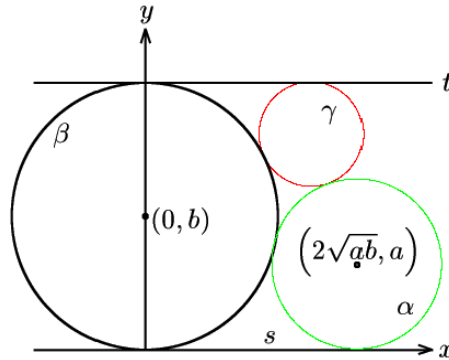


Figure 2.

We now consider the case in which the circle α is a point or a line. It is equivalent to $a = 0$ by (d2). We setup a rectangular coordinate system with origin at the point of tangency of the circle β and the line s so that

the centers of the circles β and α have coordinates $(0, b)$ and $(2\sqrt{ab}, a)$, respectively (see Figure 2). Then α has an equation

$$(2) \quad (x - 2\sqrt{ab})^2 + (y - a)^2 - a^2 = 0.$$

The equation is arranged as

$$(3) \quad \frac{x^2 + y^2}{\sqrt{a}} - 4x\sqrt{b} - 2\sqrt{a}(y - 2b) = 0,$$

and

$$(4) \quad \frac{x^2 + y^2}{a} - 4x\sqrt{\frac{b}{a}} - 2(y - 2b) = 0.$$

If $a = 0$, then the equations (2), (3), (4) imply

$$x^2 + y^2 = 0,$$

$$x = 0,$$

$$y = 2b,$$

respectively by (d1). The last three equations show that α is the origin, the y -axis, the line t , respectively. Notice that we can consider that the y -axis touches the circle β by Remark 1. Therefore the three conclusions are reasonable.

We now consider the circle γ in the same case. It has an equation $(x - 2\sqrt{bc})^2 + (y - 2b + c)^2 = c^2$. Since $c = b^2/(4a)$, the equation is arranged as

$$a(x^2 + (y - 2b)^2) - 2bx\sqrt{ab} + \frac{b^2y}{2} = 0,$$

$$\sqrt{a}(x^2 + (y - 2b)^2) - 2bx\sqrt{b} + \frac{b^2y}{2\sqrt{a}} = 0,$$

$$x^2 + (y - 2b)^2 - 2bx\sqrt{\frac{b}{a}} + \frac{b^2y}{2a} = 0.$$

If $a = 0$, the three equations give

$$y = 0,$$

$$x = 0,$$

$$x^2 + (y - 2b)^2 = 0,$$

respectively by (d1). Hence γ is the x -axis, the y -axis, the point $(0, 2b)$, respectively.

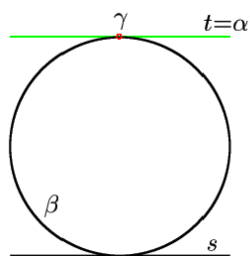


Figure 3.

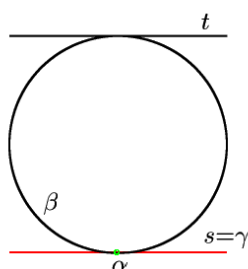


Figure 4.

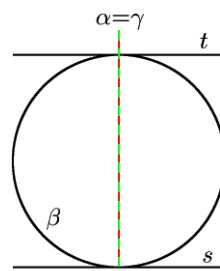


Figure 5.

If α approaches to t , then γ approaches to the point $(0, 2b)$. Therefore we can easily consider that γ is $(0, 2b)$ if α coincides with t (see Figure 3). Symmetrically γ is the line s , if α is the origin (see Figure 4). In the rest of the case, both α and γ coincide with the y -axis (see Figure 5). In all the cases the circle γ is a point or a line, i.e., $c = 0$ by (d2). Therefore (1) still holds in all the three cases.

4. CONCLUSION

Mathematics is made upon the postulates. Any postulate should be taken into consideration, if it gives reasonable conclusions with new insights. The three cases, where the circle α being a point or a line, can be obtained simply and naturally. While one of the three cases in which both of the circles α and γ coincide with the y -axis can not be obtained without the definition of the division by 0. Therefore the definition also gives us new insights of mathematics.

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